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On the non-linear analysis of the propagation of density waves through the disk of the galaxy : density and velocity perturbation

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Abstract Density waves in the galactic disk have been considered by many authors as the real cause of generation of large scale spiral arms in the disk galaxies. Propagation of the density waves through galactic disk has been analysed by various authors over the last few decades, but so far, only linear perturbations have been considered In the present paper, we have analysed the problem considering the nonlinear effects It is found that nonlinear effects are important in analysing the density wave phenomena in the outer region of the galactic disk.

Keywords : Disk galaxies, density waves, non-linear perturbation

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1. Introduction

The spectacular appearance of the spiral arms in the disk galaxies is now believed to be the results of propagation of density waves along the galactic disks. Density waves may be generated in the disk of a galaxy by gravitational disturbances in a number of ways under the influence of the differential rotation in the disk [1, 2]. The generation of density waves in such a disk has been demonstrated by numerical simulation by Lindblad [3] and Hockney [4, 5]. Lin and Shu [6, 7] and Lin [8, 9] have established the original density wave theory of Lindblad [1] on a strong theoretical foundation by invoking rigorous mathematical treatment to explain various observed phenomena in our Galaxy. These authors adopted a wave solution of the linearized gas dynamical equations and could show among other things that (a) all components of the galaxy, including the gas and the stars, should form similar spiral patterns on the scale of the radius of the disk, and that (b) the galaxies that do not show prominent spiral patterns are mostly devoid of gas and the velocity dispersions of different stellar components in them are large enough to suppress the instability completely. Basu [10] used the density wave model established by Lin and Shu [6, 7] to explain some observed phenomena in the solar neighbourhood and drew some plausible inferences of a general nature. Basu and Roy [11] extended the model to the inner region of the Galaxy and tried to interpret some of the observed dynamical behaviour of the gas in the central region. Basu *et al* [12] have derived the general dispersion relation from the wave solution of the linearized three-dimensional pressure free gas dynamical equations and deduced some useful conclusions by analyzing the density wave propagation in the *outer* and *inner* regions of the galactic disk. Similar analysis was made by Paul and Khan [13] including the pressure term in the gas dynamical equations. These latter authors found that the pressure has but minor role to influence the effects of the density wave propagation. The effects of density waves in the formation and maintenance of global spiral arms in disk galaxies have also been discussed in some details by Bertin [14] and by Toomre [15]. The fact that the density waves trigger star formation in spiral arms, has been confirmed by several authors [16-18]. Galactic magnetic field is also influenced by the propagation of density waves [19-21]. These latter authors have found among other things that the strength of the magnetic field is correlated with the strength of the density wave. This correlation again bears relation with the rate of star formation.

Thus, density waves appear to have significant influence on various aspects of the manifestation and evolution of spiral galaxies. Different authors have investigated different aspects of the effects produced by the density waves. But for such investigations, linear theories have mostly been used. The higher order effects have rarely been considered. We have therefore, considered it worthwhile to examine the higher order effects on various physical parameters which are influenced by the density wave propagation. These effects have so far remained mostly unexplored. The higher order contribution might significantly change the simple behaviour of the field variables. New Physical interpretations might emerge for the observed phenomena in the galactic disk. In order to verify these probable changes, we have undertaken the project to study and analyze the non-linear behaviour of the field variables under the density wave model of the galactic disk. In particular, we shall first consider the effect of the contribution of the second order perturbations in the field variables on various observed phenomena in the galactic disk. However, in the present paper we have discussed such effects only on the density and velocity components using two-dimensional analysis. In subsequent works, we plan to discuss other aspects of the problem including the three-dimensional analysis and additional parameters such as gas pressure and magnetic field.

2. The basic assumptions and equations

The following assumptions have been made :

(a) The effect of the galactic magnetic field is not significant and so can be ignored.

(b) The dynamics of the galactic disk can be studied by using the pressure-free gas dynamical equations.

(c) The density of gas in the disk is non-uniform and is a function of (r, θ).

(d) The rotational motion of the galactic disk is perfectly circular and the velocity is a function of the radial distance r only.

With these assumptions, the basic equations to be considered are :

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(r\rho u)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v)=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{v}{r}\left(\frac{\partial u}{\partial \theta}\right)-\frac{v^2}{r}=-\frac{\partial \phi}{\partial r}, \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+\frac{v}{r}\frac{\partial v}{\partial \theta}+\frac{uv}{r}=-\frac{1}{r}\frac{\partial \phi}{\partial \theta}, \tag{3}$$

$$\frac{\partial^2 \phi}{\partial_r 2}+\frac{1}{r}\frac{\partial \phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}=4\pi G\rho, \tag{4}$$

where ρ is the density of the gas, u and v are the radial and cross-radial components of the velocity, and ϕ is the gravitational potential satisfying Poisson's equation (4). Let the field variables ρ, u, v and ϕ are purturbed as

$$\begin{aligned}
\rho &= \rho_0+\varepsilon\rho_1+\varepsilon^2\rho_2+\ldots,\\
u &= 0+\varepsilon u_1+\varepsilon^2 u_2+\ldots,\\
v &= r\Omega(r)+\varepsilon v_1+\varepsilon^2 v_2+\ldots,\\
\phi &= \phi_0+\varepsilon\phi_1+\varepsilon^2\phi_2+\ldots,
\end{aligned} \tag{5}$$

where $\Omega(r)$ is the variable angular velocity of the disk.

3. The linear perturbation

Substituting the relations (5) in eq. (1) – (4), and equating the coefficients of ε on both sides, one gets the linear equations of perturbation in field variables, as

$$\frac{\partial \rho_1}{\partial t}+\rho_0\frac{\partial u_1}{\partial r}+\frac{\rho_0}{r}\frac{\partial v_1}{\partial \theta}+\Omega\frac{\partial \rho_1}{\partial \theta}=0, \tag{6}$$

$$\frac{\partial u_1}{\partial t}+\Omega\frac{\partial u_1}{\partial \theta}-2\Omega v_1=-\frac{\partial \phi_1}{\partial r} \tag{7}$$

$$\frac{\partial v_1}{\partial t}+\Omega\frac{\partial v_1}{\partial \theta}+\frac{K^2}{2\Omega}u_1=-\frac{1}{r}\frac{\partial \phi_1}{\partial \theta} \tag{8}$$

$$\frac{\partial^2 \phi_1}{\partial r^2}+\frac{1}{r}\frac{\partial \phi_1}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \phi_1}{\partial \theta^2}=4\pi G\rho_1. \tag{9}$$

Eq. (6) – (9) have been solved by Lin and Shu [6, 7], Lin [8, 9], Basu [10], Basu *et al* [12], Paul and Khan [13], and by others, by assuming the wave solution of the field variables in the form (assuming two-dimensional case):

$$\begin{aligned} u_1 &\equiv \exp i\,(\omega t - n\theta + K_1 r), \\ v_1 &\equiv \exp i\,(\omega t - n\theta + K_2 r), \\ \rho_1 &\equiv \exp i\,(\omega t - n\theta + K_3 r), \\ \phi_1 &\equiv \exp i\,(\omega t - n\theta + K_3 r), \end{aligned} \tag{10}$$

where ω is the wave frequency and n, K_i are respectively the wave number in θ and r-directions. The above authors have investigated the various properties of the density wave propagation through the galactic disk, using these solutions of the equations in linear perturbation. Using (10) in (6) – (9), the first order perturbed field variables are obtained as

$$u_1 = \frac{rD_1 K_3 + 2in\Omega}{rD}\phi_1, \tag{11}$$

$$v_1 = -\frac{2n\Omega D_1 - irK_3 K^2}{2r\Omega D}\phi_1, \tag{12}$$

$$\rho_1 = -\frac{\left(K_3^2 - iK_3 / r + n^2 / r^2\right)}{4\pi G}\phi_1, \tag{13}$$

where

$$\begin{aligned} D &= K^2 - (\omega - n\Omega)^2 \neq 0, \\ D_1 &= \omega - n\Omega \neq 0. \end{aligned} \tag{14}$$

Here, K is the epicyclic frequency defined by

$$K^2 = 4\Omega^2 \left[1 + \frac{r}{2\Omega}\frac{d\Omega}{dr}\right]. \tag{15}$$

Basu *et al* [12] and Paul and Khan [13] have derived the dispersion relation in the three-dimensional case and analyzed the density wave propagation in the galactic disk, in particular, in regions close to the galactic centre and far away from the centre. They have also calculated the magnitudes of the perturbation in velocity components in terms of perturbation in the density. (The latter authors [13] included the pressure term in their equations). In calculating the numerical values the authors have used the derived values given in Table 1 of Basu *et al* [12].

4. The nonlinear perturbation (second order)

Our principal aim in this paper is to make an assessment of the non-linear perturbation effects on the density wave propagation and on the resulting field variables. For the purpose we have considered here the equations with second order perturbation of the field variables. These equations are

$$\frac{\partial \rho_2}{\partial t}+\Omega\frac{\partial \rho_2}{\partial \theta}+\rho_0\frac{\partial u_2}{\partial r}+\frac{\rho_0}{r}\frac{\partial v_2}{\partial r}=\cdot\ \frac{1}{r}\frac{\partial}{\partial \Omega}(r\rho_1 u_1)-\frac{1}{r}\frac{\partial}{\partial \theta}(\rho_1 v_1), \tag{16}$$

$$\frac{\partial u_2}{\partial t}+\Omega\frac{\partial u_2}{\partial \theta}-2\Omega v_2+\frac{\partial \phi_2}{\partial r}=\frac{v_1^2}{r}-u_1\frac{\partial u_1}{\partial r}-\frac{v_1}{r}\frac{\partial u_1}{\partial \theta}, \tag{17}$$

$$\frac{\partial v_2}{\partial t}+\Omega\frac{\partial v_2}{\partial \theta}+\frac{K^2}{2\Omega}u_2+\frac{1}{r}\frac{\partial \phi_2}{\partial \theta}=\ u_1\frac{\partial v_1}{\partial r}+\frac{v_1}{r}\frac{\partial v_1}{\partial \theta}+\frac{u_1 v_1}{} \tag{18}$$

$$\frac{\partial^2 \phi}{\partial r^2}+\frac{1}{r}\frac{\partial \phi_2}{\partial r}+\frac{1}{r^2}\frac{\partial^2 \phi_2}{\partial \theta^2}=4\pi G\rho_2\,. \tag{19}$$

These equations are obtained by using the eq. (1) to (5) and retaining the perturbation quantities upto the second order. The second order field variables in perturbation will then be obtained by using eqs. (11) – (13) in eq. (16) – (19). We obtain the following relations :

$$2i\,(\omega-n\,\Omega)u_2-2\Omega\,v_2-P\phi_1^2=-2iK_3\phi_2\,, \tag{20}$$

$$(K^2/2\Omega)u_2+2i(\omega-n\Omega)v_2-Q\phi_1^2=(2in/r)\phi_2\,, \tag{21}$$

$$\rho_2=-\frac{\left(2K_3^2-iK_3/r+2n^2/r^2\right)}{2\pi G}\phi_2\,, \tag{22}$$

$$2iK_1\rho_0 u_2-2in\frac{\rho_0}{r}v_2-R\phi_1^2=\frac{2i(\omega-n\Omega)}{\pi G}\left(K_3^2-\frac{iK_3}{r}+\frac{n^2}{r^2}\right)\phi_2 \tag{23}$$

where

$$P=\ \frac{2n\Omega D_1-irK_3K^2}{2r\Omega D}\Big)'\ 1\ \ \frac{(rD_1K_3+2in\Omega)}{rD}.$$

$$\frac{iK_3(rD_1K_3+2in\Omega)}{rD}+\frac{\partial}{\partial r}\left(\frac{rD_1K_3+2in\Omega}{rD}\right)$$

$$+\frac{2n\Omega D_1-irK_3K^2}{2\Omega r^3D^2}\times(-in)\,(rD_1K_3+2in\Omega)\ =P_1+iP_2\ \text{(say)}, \tag{24}$$

$$Q=\left[\frac{(rD_1K_3+2in\Omega)}{rD}\left\{\frac{iK_3(2n\Omega D_1-irK_3K^2)}{2r\Omega D}-\frac{\partial}{\partial r}\left(\frac{2n\Omega D_1-irK_3K^2}{2r\Omega D}\right)\right\}\right.$$

$$\left.+in\left(\frac{2n\Omega D_1-irK_3K^2}{2r\Omega D}\right)^2.\frac{1}{r}+\frac{(rD_1K_3+2in\Omega)(2n\Omega D_1-irK_3K^2)}{2\Omega r^3D^2}\right]$$

$$=Q_1+iQ_2\ \text{(say)}, \tag{25}$$

$$R=-\frac{1}{r}\left[-\frac{r}{4\pi G}\left(K_3^2-\frac{iK_3}{r}+\frac{n^2}{r^2}\right)\left\{\left(\frac{rD_1K_3+2in\Omega}{rD}\right)iK_3+\right.\right.$$

$$\left.\frac{\partial}{\partial r}\left(\frac{rD_1K_3+2in\Omega}{rD}\right)\right\}-\left(\frac{rD_1K_3+2in\Omega}{rD}\right)\left\{\frac{K_3^2-\frac{iK_3}{r}+\frac{n^2}{r^2}}{4\pi G}\right.$$

$$\left.-\frac{iK_3r\left(K_3^2-\frac{iK_3}{r}+\frac{n^2}{r^2}\right)}{4\pi G}-r\frac{\partial}{\partial r}\left(\frac{K_3^2-\frac{iK_3}{r}+\frac{n^2}{r^2}}{4\pi G}\right)\right\}$$

$$\left.+\left(\frac{K_3^2-\frac{iK_3}{r}+\frac{n^2}{r^2}}{4\pi G}\right)\times\left(\frac{2n\Omega D_1-irK_3K^2}{2r\Omega D}\right)(-2in)\right]=R_1+iR_2\ \text{(say)}. \tag{26}$$

Solving for the system of eq. (20) – (23), we get the second order perturbation quantities as

$$\frac{u_2}{\rho_2}=-\frac{2\pi G}{\left(2K_3^2-\frac{iK_3}{r}+\frac{2n^2}{r^2}\right)}.\frac{X}{Z}, \tag{27}$$

and

$$\frac{v_2}{\rho_2}=-\frac{2\pi G}{\left(2K_3^2-\frac{iK}{r}+\frac{2n^2}{r^2}\right)}.\frac{Y}{Z}, \tag{28}$$

where
$$X = P\left\{-\frac{4n^2\rho_0}{r^2} - 4(\omega - n\Omega)^2\left(2K_3^2 - \frac{iK_3}{r} + \frac{2n^2}{r^2}\right)\right\}$$

$$+ Q\left\{-\frac{4K_3 n\rho_0}{r} + 2i(\omega - n\Omega)\left(2K_3^2 - \frac{iK_3}{r} + \frac{2n^2}{r^2}\right).2\Omega\right\}$$

$$+ R\left\{-4K_3(\omega - n\Omega) - \frac{4in\,\Omega}{r}\right\}, \tag{29}$$

$$Y = P\left\{\frac{4nK_1\rho_0}{r} - i(\omega - n\Omega).\frac{K^2}{\Omega}\;\frac{2K_3^2 - \frac{iK_3}{r} + \frac{2n^2}{r}}{2\pi G}\right\}$$

$$+ Q\left\{-4K_3K_1\rho_0 - 4(\omega - n\Omega)^2\left(2K_3^2 - \frac{iK_3}{r} + \frac{2n^2}{r^2}\right)\right\}$$

$$+ R\left\{\frac{4n}{r}(\omega - n\Omega) - \frac{iK_3K^2}{\Omega}\right\} \tag{30}$$

and

$$Z = P\left\{\frac{2in\rho_0K^2}{2\Omega r} - 4K_1\rho_0(\omega - n\Omega)\right\}$$

$$+ Q\left\{\frac{4n\rho_0}{r}(\omega - n\Omega) + 4iK_1\rho_0\Omega\right\} + R\left\{4(\omega - n\Omega)^2 - K^2\right\}. \tag{31}$$

5. Numerical results

For computation of numerical results in the perturbation of velocity components, both linear and second order cases, we have adopted the values of the various galactic parameters from Table 1 given in Basu *et al* [12].

5.1 Linear perturbation in velocity components :

Using the relations (11) – (13) we can write

$u_1 / \rho_1 = a_1 + ib_1$ and $v_1 / \rho_1 = a_2 + ib_2$, where

$$\frac{4\pi G}{rD}\;\frac{rD_1K_3\left(K_3^2 + n^2/r^2\right) - (2n\Omega K_3)/r}{\left(K_3^2 + \frac{n^2}{r^2}\right)^2 + \frac{K_3^2}{r^2}}$$

$$b_1 = -\frac{4\pi G}{rD} \frac{2n\Omega\left(K_3^2 + n^2/r^2\right) + D_1 K_3}{\left(K_3^2 + \frac{n^2}{r^2}\right)^2 + \frac{K_3^2}{r^2}}$$

$$a_2 = +\frac{4\pi G}{2\Omega rD} \frac{(2n\Omega D_1)\left(K_3^2 + n^2/r^2\right) + K_3^2 K^2}{K_3^2 + \cdot \quad + \frac{K_3^2}{\cdot}}$$

and

$$b_2 = +\frac{4\pi G}{2\Omega\, rD} \frac{(-rK_3K^2)\left(K_3^2 + n^2/_2\right) + \frac{1}{\cdot}(2n\Omega D_1 K_3)}{K_3^2 \quad + \frac{K_3^2}{\cdot}}$$

where K_3 is the radial wave number in density perturbation for which the adopted value by Basu *et al* [12] was $\pi\, Kpc^{-1}$. The same value has been used here. Using other values from Basu *et al* [12], the following Table 1 has been computed.

Table 1. The first order velocity perturbation relative to the perturbation in density

(Kpc)	u_1/ρ_1(Km $M_\odot^{-1}$. Sec^{-1}.Pc^{-1})	v_1/ρ_1 (Km $M_\odot^{-1}$.Sec^{-1} Pc^{-1})
1.	147.90	140 40
2	64.40	110.65
3.	42.77	13 05
4.	44.73	16 35
5.	59 20	197.59
6	83.04	233.40
7.	113.68	270.04
8.	148.96	309.71
9.	176.12	467.91
10.	207.24	391.42
11.	224.08	438.45
12.	257.86	487.22

Figures 1 (a, b) show the run of the values of u_1/ρ_1 and v_1/ρ_1 as given in Table 1. For comparison, the plot of the same parameters as obtained by Basu *et al* [12] is also presented (Figure 2 in that paper). It is found that the run of the plots are essentially similar, but the numerical values are somewhat higher in the present computation. This may be at least partially due to the difference in the adopted computation technique. It is found that the relative perturbation increases with the increasing distance from the centre of the Galaxy. But Figure 3 of Basu *et al* [12] shows that the absolute values | u_1 | and | v_1 | of the first order perturbation velocity systematically fall off with the increasing radial distance. This implies that the absolute density perturbation falls off faster with increasing radial distances. We see therefore, that the first order perturbation in physical variables generated by propagation of density waves

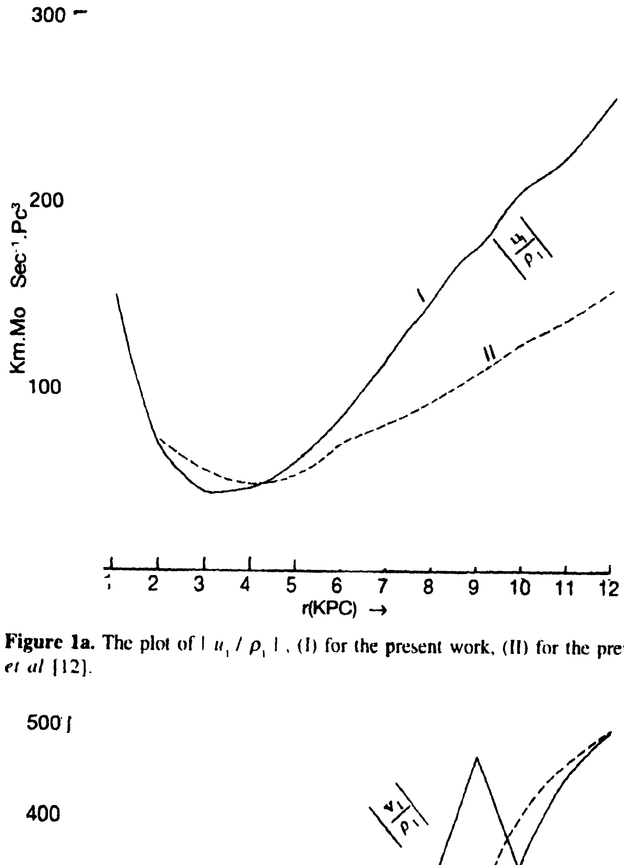

Figure 1a. The plot of | u_1 / ρ_1 |. (I) for the present work, (II) for the previous work of Basu *et al* [12].

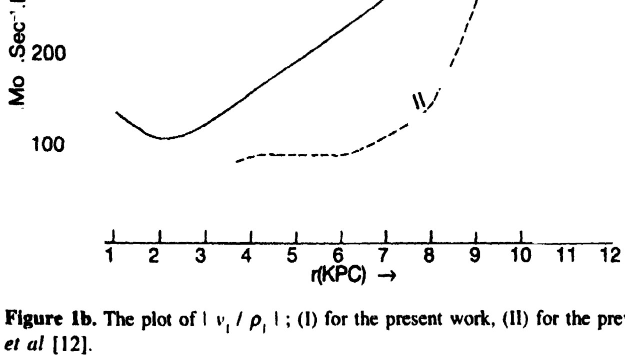

Figure 1b. The plot of | v_1 / ρ_1 | ; (I) for the present work, (II) for the previous work of Basu *et al* [12].

gradually decreases with increasing radial distance from the centre. Density waves thus form at the inner region of a galactic disk and steadily smooth out toward the periphery of the disk.

5.2 Second order perturbation in velocity components :

Using the relations (27) – (31) we can write

$$\frac{u_2}{\rho_2} = \frac{A_1 + iA_2}{B_1 + iB_2} \text{ and } \frac{v_2}{\rho_2} = \frac{C_1 + iC_2}{B_1 + iB_2},$$

where

$$A_1 = (-2\pi G)\left[-4K_3 D_1 R_1 + \frac{4n\Omega}{r} R_2 - \frac{4n\rho_0 K_3}{r} Q_1 + \frac{4\Omega D_1 K_3}{r} Q_1 - 4\Omega\, D_1 \left(2K_3^2 + \frac{2n^2}{r^2}\right) Q_2 - \frac{4n^2 \rho_0}{r^2} P_1 - 4D_1^2 \left(2K_3^2 + \frac{2n^2}{r^2}\right) P_1 + \frac{4D_1^2 K_3}{r} P_2\right],$$

$$A_2 = (-2\pi G)\left[-4K_3 D_1 R_2 - \frac{4n\Omega}{r} R_1 - \frac{4K_3 n\rho_0}{r} Q_2 + 4\Omega\, D_1 \left(2K_3^2 + \frac{2n^2}{r^2}\right) Q_1 + \frac{4\Omega D_1 K_3}{r} Q_2 - \frac{4n^2 \rho_0}{r^2} P_2 + \frac{4D_1^2 K_3}{r} P_1 - 4D_1^2 \left(2K_3^2 + \frac{2n^2}{r^2} \right| P_2 \right|,$$

$$B_1 = \left(2K_3^2 + \frac{2n^2}{r^2}\right)\left[R_1(4D_1^2 - K^2) + \frac{4n\rho_0}{r} D_1 Q_1 - 4K_1 \Omega \rho_0 Q_2 - 4K_1 \rho_0 D_1 P_1 - \frac{2n\rho_0}{r} \cdot \frac{K^2}{2\Omega} P_2\right] + \frac{K_3}{r}\left[R_2(4D_1^2 - K^2) - 4K\Omega\rho_0 Q_1 + \frac{4n\rho_0}{r} D_1 Q_2 + \frac{2n\rho_0}{r} \cdot \frac{K^2}{2\Omega} P_1 - 4K_1 \rho_0 D_1 P_2\right],$$

$$B_2 = \Big[R_2(4D_1^2 - K^2) + 4K_1\rho_0\Omega Q_1 + \frac{4n\rho_0}{-} D_1 Q_2 + \frac{2n\rho_0}{r} \cdot \frac{K^2}{2\Omega} P_1$$

$$-4K_1\rho_0 D_1 P_2\Big]\left(2K_3^2 + \frac{2n^2}{r^2}\right) - \frac{K_3}{r}\Big[R_1(4D_1^2 - K^2)$$

$$\frac{4n\rho_0}{r} D_1 Q_1 - 4K_1\rho_0\Omega Q_2 - 4K_1\rho_0 D_1 P_1 - \frac{2n\rho_0}{r} \cdot \frac{K^2}{2\Omega} P_2$$

$$C_1 = (-2\pi G)\ P_1\left(-\frac{4nK_1\rho_0}{r} - \frac{K_3}{r} \cdot \frac{K^2}{\Omega} \frac{D_1^2}{2\pi G}\right.$$

$$+P_2\left(\frac{K^2}{\Omega}\right)\left(2K_3^2 + \frac{2n^2}{r^2}\right)\frac{D_1}{2\pi G} + Q_1\Big\{-4K_3K_1\rho_0 - 4D_1^2\Big(2K_3^2 + \frac{2n^2}{r^2}$$

$$+Q_2\left(-\frac{4D_1^2K_3}{r}\right) + \frac{4nD_1}{r} R_1 + \frac{K_3K^2}{\Omega} R_2$$

$$C_2 = -(2\pi G)\ \frac{D_1K^2}{2\pi G\Omega}\left(2K_3^2 + \frac{2n^2}{r^2}\right)P_1 - \frac{4nK_1\rho_0}{r} P_2$$

$$\frac{K_3}{r} \cdot \frac{K^2}{2\pi G\Omega} D_1 P_2 + \frac{4K_3D_1^2}{r} Q_1 - 4K_3K_1\rho_0 Q_2$$

$$-4D_1^2\left(2K_3^2 + \frac{2n^2}{r^2}\right)Q_2 - \frac{K_3K^2}{\Omega} R_1 + \frac{4nD_1}{-} R_2$$

For numerical computations of u_2/ρ_2 and v_2/ρ_2, we have used the parameter values from Table 1 of Basu *et al* [12]. Also the following numerical values have been used :

n is the wave number in θ-direction and this is actually the number of spirals ; so $n = 2$ has been used ; for density wave propagation in the plane of the disk, Basu [10] obtained the radial wave numbers K_t to be equal with the numerical value $K_t = \pi\, Kpc^{-1}$; the values used for the basic density ρ_0 at different r are given in Column 6 of Table 1 of Basu *et al* [12].

With these parameter values, the numerical values of u_2/ρ_2 and v_2/ρ_2 have been computed for different radial distances from the centre, using computer. These values are given in Table 2.

A careful inspection of the values of the second order perturbation, reveals several important features. First, the second order perturbation values of u_2/ρ_2 are significantly higher than the corresponding first order perturbation values of u_1/ρ_1 . This is shown in Figure 2a.

Table 2. Values of the perturbed velocity relative to the perturbed density in the galactic disk (second order perturbations)

r (Kpc)	u_2/ρ_2(Km.$M_{\odot}^{-1}$. Sec^{-1}.Pc^{1})	v_2/ρ_2 (Km $M_{\odot}^{-1}$.Sec^{-1}.Pc^{1})
1.	32295	112.85
2	39986	11.170
3.	65870	86.077
4.	21897	1.4379
5.	57189	11.498
6	99609	51.569
7.	148831	154.19
8.	201016	348.30
9.	184081	811.23
10	282722	848.53
11.	299845	975.83
12.	337995	1250 76

Because of the incompatibility of the magnitudes of numerical values, we have shown a logarithmic plots of u_2/ρ_2 and u_1/ρ_1 in Figure 2b. The plots, however, show essentially similar trend of variations in values along the galactic disk. Figure 3 shows plots of v_1/ρ_1 and v_2/ρ_2. Here we see that v_2/ρ_2 values cover a great range. While the values of u_1/ρ_1 are greater than those of u_2/ρ_2 in the range 1-8 Kpc of the galactic disk, u_2/ρ_2 rapidly increases beyond 8 Kpc and are much higher than u_1/ρ_1. We can therefore conclude that the second order perturbation

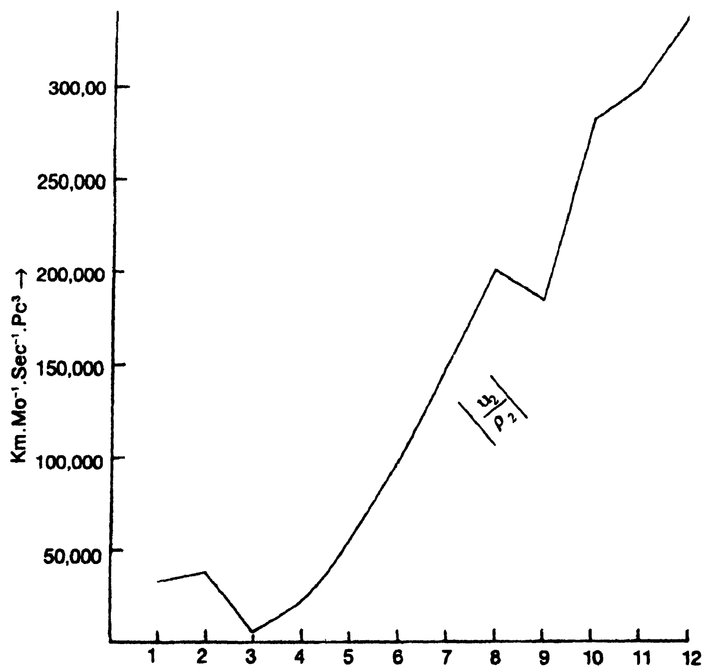

Figure 2a. The plot of | u_2 / ρ_2 | against radial distance r.

is important in analyzing the density wave phenomena in the outer region of the galactic disk. This may have a bearing on the abrupt termination of the molecular ring beyond 8 Kpc and also on the warping of the galactic disk beyond 10 Kpc. It is instructive therefore, to explore the idea that the large scale dynamics of the galactic disk will be better understood if the study is made on the basis of nonlinear analysis of the galactic dynamics. Better insight is likely to emerge

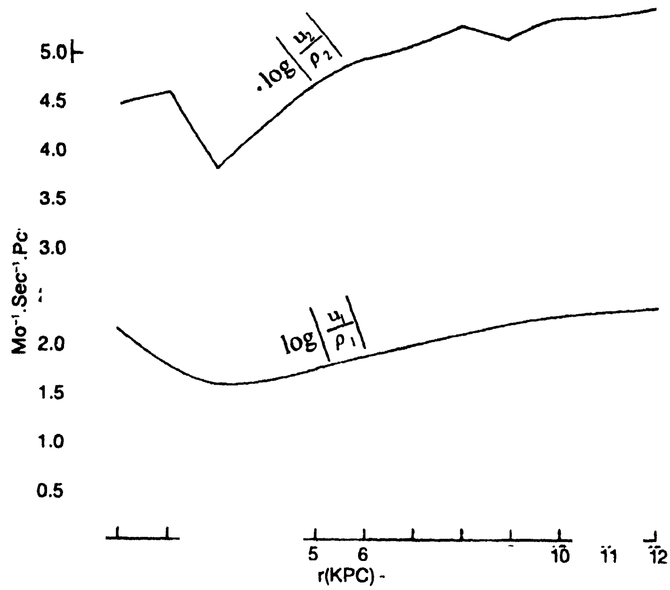

Figure 2b. The plot of $|u_1/\rho_1|$ and $|u_2/\rho_2|$ against r shown in logarithmic scale.

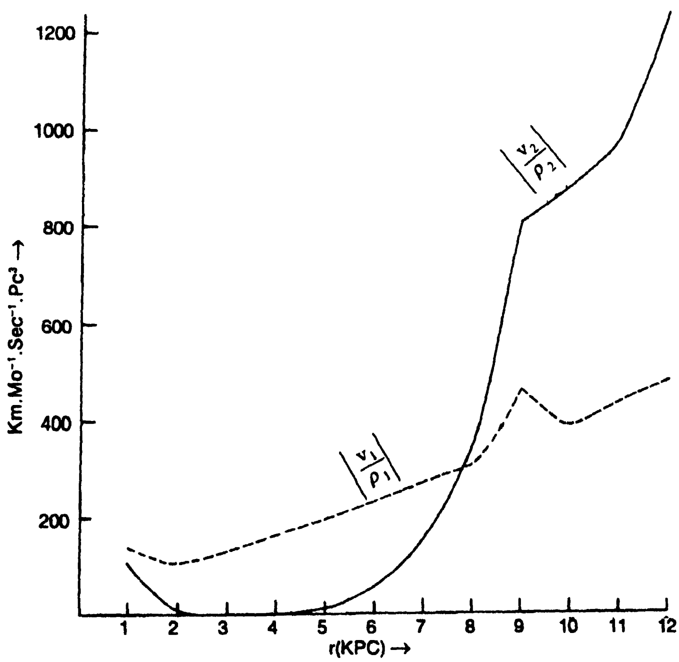

Figure 3. The plot of $|v_1/\rho_1|$ and $|v_2/\rho_2|$ against r.

when the corresponding dispersion relation is solved and analyzed. Our aim remains to do the same in a subsequent work. It may be noted that the introduction of the third dimension and the galactic magnetic field, will undoubtedly make the things a lot more complicated, but is likely to give a more complete picture of the large-scale galactic structure and dynamics.

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